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Nonlinear and Adaptive Control of Flexible Space Structures[1]

This paper addresses design of nonlinear control systems for rapid, large angle multiaxis, slewing and LOS pointing of realistic flexible space structures. The application of methods based on adaptive feedback linearization for nonlinear control design for flexible space structures is presented. A comprehensive approach to modeling the nonlinear dynamics and attitude control of multibody systems with structural flexure is considered. Adaptive feedback linearizing control laws are described based on Lagrangian dynamical system model for the spacecraft. Simulation results for attitude slewing and LOS stabilization for the NASA/IEEE Spacecraft COntrols Laboratory Experiment (SCOLE) design challenge are presented.

Introduction

An important class of spacecraft missions involve deployed payload apertures attached to cantilevered flexible appendages. Dynamic response of attitude control and LOS pointing for such systems can involve nonlinear control structure interaction. A benchmark problem for slewing and pointing control of a realistic flexible spacecraft is provided by the NASA/IEEE SCOLE design challenge (Taylor and Balakrishnan, 1984). The important features for the SCOLE problem include nonlinear attitude response during large angle, multiaxis rotation of a primary body (shuttle orbiter) with elastic deformations of an attached appendage. LOS pointing of an rf subsystem involves the relative motions of the prime body and a reflector attached to the tip of the attached appendage. Numerous modeling and control design studies for SCOLE have been reported which address the linear dynamics and control of the appendage flexure (Fisher, 1989). Slewing maneuver control design methods for spacecraft with significant structural flexure are generally available only under special constraints such as planar maneuvers (Singh et al., 1989). This fact underlies the importance of nonlinear dynamics for such applications. Kakad (1987) develops dynamic equations of motion for the large angle, multiaxis attitude dynamics of SCOLE. Flexure response of the appendage system is modeled using a finite element expansion based on assumed modes. Nonlinear control design for large angle slewing of the SCOLE system has been previously reported (Azam et al., 1990). However, nonlinear control design methods of this type are model-based and function by decoupling of critical nonlinear interactions to achieve closed loop control. Successful application of nonlinear model-based control design methods to flexible space structures will require consideration for robustness to model uncertainty. Design for enhanced robustness in nonlinear control systems can benefit from careful integration of the modeling and control design to represent model uncertainty. In this paper we address these issues for the SCOLE design challenge and provide evidence of robust performance for nonlinear control design for attitude and LOS pointing control.

In this paper we consider the application of the modeling and control design methods reported in Kwatny and Bennett (1988) and Bennett et al. (1990b) and discuss a realistic benchmark problem in detail. We first describe a comprehensive approach to modeling of multibody systems with elastic structural interactions based on the perspective of Lagrange's equations and Hamilton's principle. Structural flexure is modeled as a distributed parameter system using a spatially continuous Lagrangian. Spatial discretization for model reduction is described using finite element approximation by collocation using B-splines. We next describe the design of nonlinear Partial Feedback Linearizing (PFL) compensation for decoupling, linearization, and integration of large angle slewing control with active structural control. PFL methods involve nonlinear, model-based compensation of system nonlinearities and are therefore subject to robustness concerns. The approach we develop for slewing and pointing employs a modified Model Reference Adaptive Control (MRAC) scheme for PFL attitude control which improves control system robustness.

The development of a PFL control law for SCOLE reported in this paper differs from the approach considered in Azam et al. (1990) in that the control derivation is based on the explicit structure of the dynamics arising from Lagrange's equations. This simplifies the derivation of the nonlinear control laws and clarifies several important technical issues related to feedback linearizing control methods as applied to flexible space structures. The modeling approach also facilitates the introduction of MRAC for enhanced robustness of the PFL attitude control laws. Indeed, the Lagrangian formulation described

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Table 1 Simulation parameters for SCOLE model

Parameter	Value	Explanation
E	3.0E9	modulus of elasticity
μ	0.28	Poisson's ratio
ρ	2.2768E4	mass density of mast
A	6.955E-4	cross sectional area
m_R	181.4	mass of reflector
l	39.6	length of mast
$I_{S,xx}$	1.23E6	inertia matrix for shuttle
$I_{S,xz}$	−1.97E5	
$I_{S,yy}$	9.20E6	
$I_{S,zz}$	9.60E6	
$I_{R,xx}$	2.44E4	inertia matrix for reflector
$I_{R,xy}$	1.03E4	
$I_{R,yy}$	1.27E4	
$I_{R,zz}$	3.72E4	

Table 2 Notation for SCOLE model

Notation	Explanation
A^T	transpose of matrix A
$\dot{x} = dx/dt$	time differentiation
$x_z(t, z) = \partial x/\partial z\ (z, t)$	partial differentiation
Appendage Deformation Coordinates ($0 \le z \le l$)	
$\eta_1(z)$	appendage lateral deformation (along x-axis)
$\eta_2(z)$	appendage lateral deformation (along y-axis)
$\xi^T(z) = (\psi, \theta, \phi)$	appendage angular (rotational shear) deformation (3-2-1 convention)
$\psi(z)$	about z-axis (yaw)
$\theta(z)$	about y-axis (pitch)
$\phi(z)$	about x-axis (roll)
Rigid Body Coordinates	
$\gamma_S = (\gamma_1, \gamma_2, \gamma_3)^T$	Gibbs vector for Shuttle attitude
ω_S	angular rates of Shuttle fixed frame
$\eta_R = \eta(l) \in \Re^2$	lateral translation of Reflector in Shuttle body frame
$\gamma_R = \xi(l) \in \Re^3$	rotation of Reflector frame relative to Shuttle frame
Rigid Body Parameters	
m_R	mass of Reflector
I_R	Reflector inertia tensor (about point of attachment)
I_S	Shuttle inertia tensor
Appendage Effective Beam Parameters	
l	length of mast
ρ	mass density
A	cross section area
E	elasticity
κG	effective shear modulus
I	area moment of inertia
External Control Inputs	
τ_S	three component external torques applied about shuttle frame
τ_R	three component external torques applied about reflector frame
f_R	two component external forces applied to reflector

herein assures that the system is feedback linearizable for all admissable values of the uncertain system parameters. This is a key requirement of the adaptive control laws used to affect robust feedback linearization. The use of MRAC to enhance robustness of PFL control design has been previously described in Taylor et al. (1989) and with application to control of flexible space structures in Akhrif (1989). The application of nonlinear MRAC for SCOLE has not been reported previously.

Flexible Spacecraft Attitude Dynamics

The SCOLE system is a simple but illustrative example of the modeling issues which arise in flexible space structures modeled as multibody systems with flexible interactions; i.e., *multi-flex-body systems*. The slewing and alignment dynamics

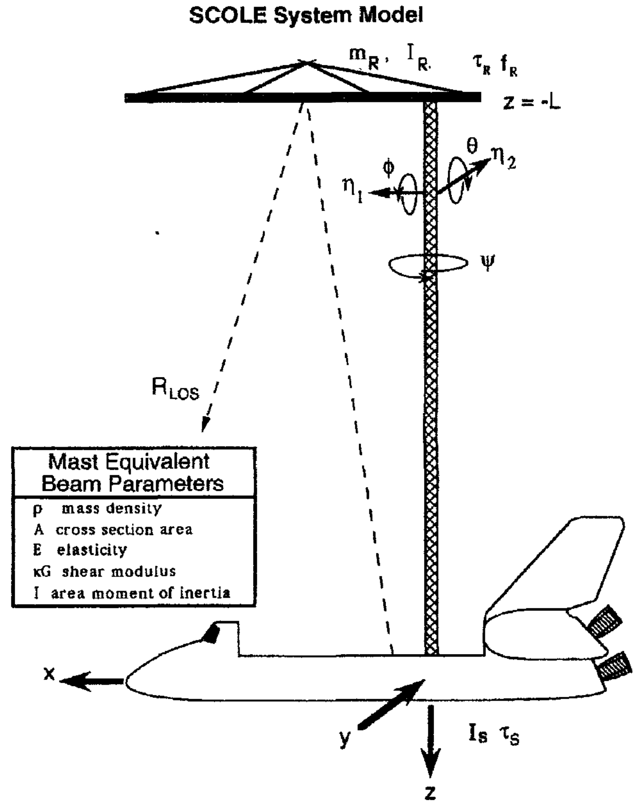

Fig. 1 SCOLE system configuration

of the mast/reflector system involves shuttle attitude reorientation maneuvers with provisions for active structural (alignment) control of the mast. We employ a Lagrangian approach to model formulation which facilitates the computation of decoupling and feedback linearizing control laws for coordination of slewing and LOS stabilization. Notation is summarized in Table 2.

The SCOLE control problem definition considered in this paper is taken from Taylor and Balakrishnan (1984). The SCOLE system is depicted in Fig. 1. The control inputs include three independent torques applied at the shuttle τ_S, three independent torques applied about the reflector τ_R, and two independent forces in the X and Y direction applied at the reflector point of attachment to the mast f_R (see Table 2).

Hamilton's Principle and the Euler-Lagrange Equations. The formalism of Lagrangian dynamics begins with the identification of the configuration space, i.e., the generalized coordinates, associated with the dynamical system of interest. Once the configuration manifold, $\mathfrak{M}$, is specified we have the natural definition of velocity at a point $q \in \mathfrak{M}$ as a vector, $\dot{q}$, in the tangent space to $\mathfrak{M}$ at q, often denoted $T_q\mathfrak{M}$. The state space for the evolution model is the tangent bundle $T\mathfrak{M}$ (Abraham and Marsden, 1978; Arnold, 1978). The evolution dynamics of the system is characterized using Hamilton's principle of least action by the definition of a Lagrangian $L(q, \dot{q})$: $T\mathfrak{M} \rightarrow \Re$. Hamilton's principle characterizes the "natural" motion in the presence of external generalized forces, Q, via the variational relation

$$\int_{t_1}^{t_2} (\delta L + Q^T \delta q)\, dt = 0. \tag{1}$$

For *Distributed Parameter Systems* (*DPS*), special consideration is required to properly characterize the configuration space. The approach followed herein is to choose the generalized coordinates such that all "nonworking" or geometric constraints on the motion are eliminated (Baillieul and Levi, 1987). This is the key to the utility of the Lagrange formalism for constructing the equations of motion. Any "geometric" boundary conditions (which we will denote $\mathcal{G}$) are therefore included as part of the definition of the configuration space. Any additional boundary conditions required, in conjunction with the Euler-Lagrange equations, arise naturally from Hamilton's Principle, Eq. (1), and are referred to as "natural" boundary conditions (denoted $\mathfrak{N}$). We denote by H^p the completion of the set of functions with p continuous derivatives and which satisfy

$$\|v\|_p^2 = \int_0^l \{|D^p v(z)|^2 + \ldots + |v(z)|^2\} dz < \infty \quad (2)$$

These are the *Sobolev spaces* (Lions, 1971).

Let $H_{\mathcal{G}}^p$ denote the completion of the set of functions satisfying Eq. (2) as well as a prescribed set of (geometric) boundary conditions designated $\mathcal{G}$. It is not necessarily true that all of the functions in this new space satisfy the boundary conditions. The reason for this is that an arbitrary sequence of functions, all satisfying the given boundary conditions, may converge to a function which does not satisfy the boundary conditions. However, the following proposition is true. Suppose the boundary conditions $\mathcal{G}$ involve derivatives of order s and none higher. Then all of the functions in $H_{\mathcal{G}}^p$ satisfy the boundary conditions provided $p > s$. Thus, a consistent definition of the configuration space is obtained *if the specified norm is compatible with the geometric boundary conditions.*

Hamilton's principle may be used to derive the Euler-Lagrange equations and the natural boundary conditions. The Euler-Lagrange equations are to be solved along with boundary conditions $\mathcal{B} = \mathcal{G} \cup \mathfrak{N}$. These solutions are "strong" solutions of Hamilton's principle (1). In general, the Lagrangian will involve derivatives with respect to z of order p and the Euler-Lagrange equations will involve derivatives of order $2p$. For application to flexible structures we are usually interested in numerical approximation of the "weak" (sometimes called generalized or distributional (Strang and Fix, 1973; Stakgold, 1979)) solutions in $H_{\mathcal{G}}^p$ which satisfy Hamilton's principle, but do not necessarily have the same degree of differentiability as the strong solutions.

Finite Dimensional Approximation. Finite dimensional approximations to the evolution dynamics of Lagrangian systems may be derived from either the Euler-Lagrange equations or Hamilton's principle. The latter is the basis for the Finite Element Method (FEM) described in (Strang and Fix, 1973). The models developed in this study use FEM reduction of the system Lagrangian based on *collocation by splines* (Stakgold, 1979).

Kinematics of Attitude Slewing and Pointing. The SCOLE slewing problem with structural flexure is adequately modeled by including dynamic degrees-of-freedom for attitude motions of the base body (shuttle orbiter) and flexure of the appendage or mast. Thus we take as a configuration space for the SCOLE slewing model, $q \in SO(3) \times H_{\mathcal{G}}^p$, where $q = \{L, \eta(z), \xi(z)\}$, $L \in SO(3)$, and $H_{\mathcal{G}}^p$ is the space of functions, continuously differentiable to first order, defined on the interval $z \in [0, l]$, and which satisfy the geometric boundary conditions; $\eta(0) = 0$, $\xi(0) = 0$. The mass of the mast/reflector system for SCOLE is a relatively small fraction of the total system mass and we assume that deformations of the mast in the shuttle fixed frame are small enough so that CG movement is negligible. We also assume that mast longitudinal deformation can be neglected. (See Figure 1.)

A convenient choice of coordinates for $SO(3)$ describing the attitude of the shuttle body frame is the Gibbs vector (Wertz et al., 1978); $\gamma_s \in \mathbb{R}^3$. The generalized coordinates for the SCOLE system can be given via this parameterization locally as, $q := \{\gamma_S, \eta(z), \xi(z)\}$. A natural choice for quasi-velocities (i.e., generalized velocities) is: $p := \{\omega_S, \dot{\eta}(z), \dot{\xi}(z)\}$. Attitude kinematics of the shuttle-mast-reflector system are then readily expressed as:

$$\dot{\gamma}_S(t) = \Gamma(\gamma_S)\omega_S$$

where (Wertz et al., 1978)

$$\Gamma(\gamma_S) := \frac{1}{2}[I_3 + \gamma_S\gamma_S^T + \Omega(\gamma_S)] \quad (3)$$

where the skew symmetric matrix

$$\Omega(x) := \begin{bmatrix} 0 & -x_3 & x_2 \\ x_3 & 0 & -x_1 \\ -x_2 & x_1 & 0 \end{bmatrix},$$

will be represented in the sequel as $\Omega_S = \Omega(\omega_S)$.

Kinetic Energy Expression Using Quasi-Coordinates. The DPS kinetic energy expression for the SCOLE system can be written in component form:

$$T = T_{\text{shuttle}} + T_{\text{flex}_1} + T_{\text{flex}_2} + T_{\text{reflector}} \quad (4)$$

where

$$T_{\text{shuttle}} = \frac{1}{2}\omega_S^T I_S \omega_S, \quad (5)$$

$$T_{\text{flex}_1} = \frac{1}{2}\int_0^l \|\Omega_S\eta(z) + \dot{\eta}(z)\|^2 \rho A dz, \quad (6)$$

$$T_{\text{flex}_2} = \frac{1}{2}\int_0^l [\omega_S + \dot{\xi}(z)]^T I [\omega_S + \dot{\xi}(z)]\rho dz \quad (7)$$

$$T_{\text{reflector}} = \frac{1}{2} m_R \|\Omega_S\eta_R + \dot{\eta}_R\|^2 + \frac{1}{2}\{\omega_S + \dot{\xi}_R\}^T I_R \{\omega_S + \dot{\xi}_R\} \quad (8)$$

The simple form of the kinetic energy expression follows from the choice of quasi-coordinates and the geometric boundary conditions for the spatially distributed mast flexure.

FEM Reduction via Collocation by Splines. Collocation methods are a popular form of Galerkin approximation in finite element analysis which is useful in model reduction and simulation of elastic response of structures. In these methods the coordinates are physical deformations at fixed node points. For homogeneous structures in one spatial dimension, splines offer a simple and practical approach to collocation which is useful in approximating the weak solutions of the DPS. This approach provides a FEM expansion for each of the deformation coordinates of the form:

$$\eta_i(z,t) \approx \Phi^T(z)\bar{\eta}_i(t) \text{ for } i = 1,2 \quad (9)$$

$$\xi_j(z,t) \approx \Phi^T(z)\bar{\xi}_j(t) \text{ for } j = 1,2,3 \quad (10)$$

where $\bar{\eta}_i$ and $\bar{\xi}_j$ are each N-vectors. The reduced FEM coordinates of mast deformation are: $\bar{\eta}^T = [\bar{\eta}_1^T, \bar{\eta}_2^T]$ of dimension $2N$ and $\bar{\xi}^T = [\bar{\phi}^T, \bar{\theta}^T, \bar{\psi}^T]$ of dimension $3N$.

Reduction of the Kinetic Energy Function. To reduce the kinetic energy expression we obtain the total system kinetic energy in the standard form,

$$T \approx \frac{1}{2}p^T M(q) p, \quad (11)$$

using the order N FEM expansions, Eqs. (9)–(10). Expressions for the system mass matrix can be directly obtained by substituting the expansions (9)–(10) into the DPS kinetic energy expressions (6)–(7). The reduced kinetic energy terms take the form:

$$T_{\text{flex}_1} \approx \frac{1}{2}\{\omega_S^T I_{\omega\omega}(\bar{\eta})\omega_S + 2\omega_S^T I_{\omega\eta}(\bar{\eta})\dot{\bar{\eta}} + \dot{\bar{\eta}}^T I_{\eta\eta}\dot{\bar{\eta}}\} + \frac{1}{2} m_R \|\Omega_S \eta_l + \dot{\eta}_l\|^2 \quad (12)$$

$$T_{\text{flex}_2} \approx \frac{1}{2}\{\omega_S^T J_{\omega\omega}\omega_S + 2\omega_S^T J_{\omega\xi}\dot{\bar{\xi}} + \dot{\bar{\xi}}^T J_{\xi\xi}\dot{\bar{\xi}}\} \quad (13)$$

where $\xi_l = \xi(l)$, and the "mass" matrix takes the form:

$$M(\gamma_S, \bar{\eta}, \bar{\xi}) = \begin{bmatrix} M_\omega & N \\ N^T & M_u \end{bmatrix} = \left[\begin{array}{c|cc} I_S + I_R + I_{\omega\omega}(\bar{\eta}) + J_{\omega\omega} & I_{\omega\eta}(\bar{\eta}) & J_{\omega\xi} \\ \hline I_{\omega\eta}^T(\bar{\eta}) & I_{\eta\eta} & 0 \\ J_{\omega\xi}^T & 0 & J_{\xi\xi} \end{array}\right]. \quad (14)$$

The expressions for the system inertia terms are summarized in an appendix.

Reduction of the Potential Energy and Dissipation Functions. For modeling the SCOLE slewing and LOS pointing dynamics recall that structural deformations of the mast are assumed small in the shuttle-fixed frame. Thus the potential energy expression for the elastic continuum is assumed to have the form:

$$V = \frac{1}{2}\int_0^l \{\xi_z^T \tilde{K}_e \xi_z + (P\eta_z - \xi)^T \tilde{K}_s (P\eta_z - \xi)\} dz$$

where

$$P^T := \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix},$$

$\tilde{K}_e = \text{diag}(GJ, EI_{yy}, EI_{zz})$, and $\tilde{K}_s = \text{diag}(\kappa_1 GA, \kappa_2 GA, \mu EA)$; i.e., we express the structural elastic stored energy in terms of strains. Using the FEM approximation to reduce the potential energy obtains the form:

$$V \approx \frac{1}{2}\bar{\eta}^T K_{\eta\eta}\bar{\eta} + \frac{1}{2}\bar{\xi}^T K_{\xi\xi}\bar{\xi} + 2\bar{\eta}^T K_{\eta\xi}\bar{\xi} = \frac{1}{2} q^T \begin{bmatrix} 0 & 0 \\ 0 & K_{\text{mast}} \end{bmatrix} q \quad (15)$$

where the generalized coordinates are as above and

$$K_{\text{mast}} = \begin{bmatrix} K_{\eta\eta} & K_{\xi\eta}^T \\ K_{\xi\eta} & K_{\xi\xi} \end{bmatrix}. \quad (16)$$

A model for elastic dissipation compatible with the assumptions for the potential energy can be obtained by consideration of a dissipation function of the form:

$$R(\dot{\eta}, \dot{\xi}) = \frac{1}{2}\int_0^l \{\dot{\eta}^T \Xi_1 \dot{\eta} + \dot{\xi}^T \Xi_2 \dot{\xi} + (\dot{\eta}_z)^T \Xi_3 \dot{\eta}_z + (\dot{\xi}_z)^T \Xi_4 (\dot{\xi}_z)\} dz \quad (17)$$

where Ξ_i, $i = 1, 2$ (resp. $i = 3, 4$) can be chosen to model viscous (e.g., external) damping effects (resp. internal, strain rate dissipation). Using the FEM expansions, the dissipation function reduces to the form:

$$R \approx \frac{1}{2}\dot{\bar{\eta}}^T B_{\eta\eta}\dot{\bar{\eta}} + \frac{1}{2}\dot{\bar{\xi}}^T B_{\xi\xi}\dot{\bar{\xi}} + 2\dot{\bar{\eta}}^T B_{\eta\xi}\dot{\bar{\xi}} = \frac{1}{2} p^T \begin{bmatrix} 0 & 0 \\ 0 & B_{\text{mast}} \end{bmatrix} p \quad (18)$$

with generalized velocities, p, and

$$B_{\text{mast}} = \begin{bmatrix} B_{\eta\eta} & B_{\xi\eta}^T \\ B_{\xi\eta} & B_{\xi\xi} \end{bmatrix}. \quad (19)$$

The appendix contains the expressions obtained from FEM expansions of the potential energy and dissipation functions.

Euler-Lagrange Equations in Quasi-Velocities. The Euler-Lagrange equations for the SCOLE system, as derived from Hamilton's principle with the reduced Lagrangian, are:

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{\gamma}_S} - \frac{\partial L}{\partial \gamma_S} = Q_{\gamma S}, \quad (20)$$

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{\bar{\eta}}} - \frac{\partial L}{\partial \bar{\eta}} + \frac{\partial R}{\partial \dot{\bar{\eta}}} = Q_\eta, \quad (21)$$

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{\bar{\xi}}} - \frac{\partial L}{\partial \bar{\xi}} + \frac{\partial R}{\partial \dot{\bar{\xi}}} = Q_\xi, \quad (22)$$

where

$$L = T - V = \frac{1}{2} p^T M(q) p - \frac{1}{2} q^T K q, \quad (23)$$

and the generalized forces are defined in terms of the *virtual work expression*;

$$\delta W = Q_{\gamma_S}^T d\gamma_S + Q_\eta^T d\eta + Q_\xi^T d\xi. \quad (24)$$

The notion of quasi-velocities and quasi-coordinates leads to a convenient form of Lagrange's equations which is applicable to systems with nonholonomic as well as the usual holonomic constraints. Such generalizations were produced at the turn of the century and are associated with the names of Poincare, Appell, Maggi, Hamel, Gibbs and Boltzman (Arnold et al., 1988). A variant of these formulations has been recently popularized by Kane and Levinson (1985). To obtain the SCOLE equations of motion using the quasi-velocity dependent system Lagrangian expression we make use of the following lemma.

Lemma. Given a system Lagrangian $L(\omega)$ where ω is a quasi-velocity related to the coordinate γ via the relation,

$$\omega = \Sigma(\gamma)\dot{\gamma}$$

then

$$\left[\frac{d}{dt}\frac{\partial L}{\partial \dot{\gamma}} - \frac{\partial L}{\partial \gamma}\right]^T = \Sigma^T(\gamma)\left[\frac{da}{dt} + \omega \times a\right] \quad (25)$$

where $a = (\partial L/\partial\omega)^T$. (See also, Baillieul and Levi, 1987; Bennett et al., 1990b; Arnold et al., 1988.) □

Thus (20) can be replaced using quasi-velocities with,

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{\omega}_S} + \omega \times \frac{\partial L}{\partial \omega_S} = \Gamma^T Q_{\gamma S}, \quad (26)$$

and direct construction of Lagrange's equations (applying the kinematic definitions) yields the equations of motion in the form:

$$M(q)\dot{p} + \left[\frac{\partial M(q)p}{\partial q}\right]p - \frac{1}{2}\left[\frac{\partial M(q)p}{\partial q}\right]^T p + \begin{bmatrix} 0 & 0 \\ 0 & B_{\text{mast}} \end{bmatrix} p + \begin{bmatrix} 0 & 0 \\ 0 & K_{\text{mast}} \end{bmatrix} q = \begin{pmatrix} \Gamma^T Q_{\gamma S} \\ Q_\eta \\ Q_\xi \end{pmatrix}. \quad (27)$$

Available control forces acting on the SCOLE system are given in Table 2. It is easy to show that the virtual work expression (24) involves generalized forces given as:

$$Q_\gamma = \Gamma^{-T}\{\tau_S + \tau_R - Z_l f_R\}$$

$$Q_\eta = [0_{2\times 2N-2}, I_2]^T f_R$$

$$Q_\xi = [0_{3\times 3N-3}, I_3]^T \tau_R.$$

Finally, a straightforward computation obtains the SCOLE equations of motion in the form:

$$\dot{\gamma}_S = \Gamma(\gamma_S)\omega_S \tag{28}$$

$$M(q)\dot{p} + B(q,p)p + K(p,q)q = G_S\tau_S + G_R\tilde{f}_R \tag{29}$$

where $\dot{f}_R^T = [f_R^T, \tau_R^T]$, $M(q)$ is given in (14),

$$G_S = \begin{bmatrix} I_3 \\ 0_{2N\times 3} \\ 0_{3N\times 3} \end{bmatrix}, G_R := \left[\begin{array}{c|c} Z_l & I_3 \\ \hline 0_{2N-2\times 2} & \\ & 0_{2N} \\ I_2 & \\ \hline & 0_{3N-3\times 3} \\ 0_{3N\times 2} & \\ & I_3 \end{array}\right], \tag{30}$$

$$Z_l := \begin{bmatrix} 0 & -l & 0 \\ l & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

$$B(q,p) = \begin{bmatrix} B_{\omega\omega} & B_{\omega u} \\ B_{u\omega} & B_{uu} \end{bmatrix}$$

$$:= \left[\begin{array}{c|cc} \Omega_S\{I_S + I_R + I_{\omega\omega}(\bar{\eta}) + J_{\omega\omega}\} & \Omega_S I_{\omega\eta}(\bar{\eta}) + H & \Omega_S J_{\omega\xi} \\ \hline -\frac{1}{2}H^T & W + B_{\eta\eta} & B_{\eta\xi} \\ 0_{3N\times 3} & B_{\eta\xi}^T & B_{\xi\xi} \end{array}\right] \tag{31}$$

$$K(p,q) := \begin{bmatrix} 0 & 0 \\ 0 & K_{\text{mast}} \end{bmatrix}, \tag{32}$$

$$H(\bar{\eta},\omega_S,\dot{\bar{\eta}}) := \frac{\partial}{\partial\bar{\eta}}\{I_{\omega\omega}(\bar{\eta})\omega_S + I_{\omega\eta}(\eta)\dot{\bar{\eta}}\} \tag{33}$$

$$= \begin{bmatrix} -\omega_2\bar{\eta}_2^T N_{\eta\eta} - \omega_3 N_\eta^T & 2\bar{\eta}_2^T N_{\eta\eta}\omega_1 - \omega_2\bar{\eta}_1^T N_{\eta\eta} \\ -\omega_1\bar{\eta}_2^T N_{\eta\eta} + 2\omega_2\bar{\eta}_1^T N_{\eta\eta} & -\omega_1\bar{\eta}_1^T N_{\eta\eta} - \omega_3 N_\eta^T \\ -\omega_1 N_\eta^T + 2\omega_3\bar{\eta}_1^T N_{\eta\eta} + \dot{\bar{\eta}}_2^T N_{\eta\eta} & -\omega_2 N_\eta^T + 2\omega_3\bar{\eta}_2^T N_{\eta\eta} - \dot{\bar{\eta}}_1^T N_{\eta\eta} \end{bmatrix}$$

$$W(\omega_S,\bar{\eta}) := \frac{\partial}{\partial\bar{\eta}}\{I_{\omega\eta}^T(\bar{\eta})\omega_S\} = \begin{bmatrix} 0 & -\omega_3 N_{\eta\eta} \\ \omega_3 N_{\eta\eta} & 0 \end{bmatrix}. \tag{34}$$

Attitude Control Design

The approach considered for attitude slewing and LOS stabilization is based on feedback linearization of the attitude response of the shuttle orbiter using torques applied to the shuttle (base body). This could be implemented by the onboard shuttle attitude control system. The control laws result in a nonlinear feedback compensation which decouples the shuttle attitude response from the flexure response of the mast/reflector system and attitude slewing is obtained using shuttle attitude control system. The mast/reflector system is then stabilized using feedback from collocated deformation measurements to force and torque actuators applied at the mast tip. (This control actuation architecture is consistent with the design challenge in Taylor and Balakrishnan (1984)). The control scheme is made robust to uncertain structure dynamics through the introduction of a modified Model Reference Adaptive Control (MRAC) for PFL attitude control.

The system dynamic equations for SCOLE attitude developed in section 2 are an instance of a class of Lagrangian systems expressed in terms of quasi-velocities, in which the control objective is given in terms of position coordinates. The attitude control dynamics can be written in the form,

$$M_\omega\dot{\omega} + N\ddot{u} + \phi_\omega = G_\omega\tau \tag{35}$$

$$M_u\ddot{u} + N^T\dot{\omega} + \phi_u = G_u\tau \tag{36}$$

with τ an m-vector of (torque) controls and ω an m-vector of quasi-velocities. The objective is to regulate an m-vector of position coordinates (e.g., attitude parameters), y (resp. $y = \gamma$), which are related to the quasi-velocities via a kinematic relation of the form,

$$\dot{\gamma} = \Gamma(\gamma)\omega. \tag{37}$$

The concept of *Partial Feedback Linearization* (*PFL*) offers a general approach to the design of nonlinear control systems for a rather general class of nonlinear systems with smooth nonlinearities (Isidori, 1985). Attitude control of rigid spacecraft using feedback linearization was first considered by Dwyer (1984). In this paper, we consider PFL control laws for attitude control of spacecraft with flexible structures and focus attention on the structure of these control laws for such systems. We show how certain natural considerations on the choice of actuators and their location on the structure simplify the PFL laws for attitude control.

A PFL compensation for the Lagrangian system (35)–(36) is a nonlinear feedback control of the form,

$$\tau = \mathfrak{A}(\gamma,\dot{\gamma},u,\dot{u}) + \mathfrak{B}(\gamma,\dot{\gamma},u,\dot{u})\alpha \tag{38}$$

which renders the compensated (i.e., closed loop) attitude response γ from the synthetic input commands α in linear, decoupled form:

$$\ddot{\gamma} = \alpha. \tag{39}$$

Specific conditions for the existence and construction of such control laws for a significantly more general class of nonlinear systems are given in Isidori (1985). In the following we extend the discussion in Bennett et al. (1990a) on the construction of PFL control for attitude control of flexible structures with emphasis on the use of quasi-coordinates.

Proposition. The PFL control for regulation of spacecraft attitude parameterized by the Gibbs vector, $y = \gamma$ in (37), for the system equations (35)–(36) takes the form (38) with

$$\mathfrak{A} = [G_\omega - NM_u^{-1}G_u]^{-1}\{\phi_\omega - NM_u^{-1}\phi_u + [NM_u^{-1}N^T - M_\omega]y^T\omega\omega\} \tag{40}$$

$$\mathfrak{B} = [G_\omega - NM_u^{-1}G_u]^{-1}[NM_u^{-1}N^T - M_\omega]\Gamma^{-1} \tag{41}$$

Proof. The first step is to transform the model equations to coordinate velocities. Computation of the transform is simplified by the introduction of a Gibbs vector for attitude parametrization. It is shown in Bennett et al. (1990b) that with Γ as defined in (3)

$$\ddot{\gamma} = \gamma^T\omega\Gamma(\gamma)\omega + \Gamma(\gamma)\dot{\omega}.$$

Thus solving for $\dot{\omega}$ and substituting into (35)–(37) obtains the equations in the transformed coordinates as,

$$\begin{bmatrix} \tilde{M}_{11} & \tilde{M}_{12} \\ \tilde{M}_{21} & \tilde{M}_{22} \end{bmatrix}\begin{pmatrix} \ddot{\gamma} \\ \ddot{u} \end{pmatrix} + \begin{pmatrix} \phi_\omega - M_\omega\gamma^T\omega\omega \\ \phi_u - N^T\gamma^T\omega\omega \end{pmatrix} = \begin{pmatrix} G_y \\ G_u \end{pmatrix}\tau \tag{42}$$

where $\tilde{M}_{11} = M_y\Gamma^{-1}$, $\tilde{M}_{21} = N^T\Gamma^{-1}$, $\tilde{M}_{12} = N$, $\tilde{M}_{22} = M_u$. To identify the PFL control we solve for the accelerations:

$$\begin{pmatrix} \ddot{y} \\ \ddot{u} \end{pmatrix} = \begin{pmatrix} \Sigma_y \\ \Sigma_u \end{pmatrix} + \begin{pmatrix} Q_y \\ Q_u \end{pmatrix}\tau,$$

$$= \begin{bmatrix} \tilde{M}_{11} & \tilde{M}_{12} \\ \tilde{M}_{21} & \tilde{M}_{22} \end{bmatrix}^{-1}\left\{\begin{pmatrix} G_y \\ G_u \end{pmatrix}\tau - \begin{pmatrix} \phi_\omega - M_\omega\gamma^T\omega\omega \\ \phi_u - N^T\gamma^T\omega\omega \end{pmatrix}\right\},$$

then the PFL (decoupling) control law is:

$$\tau = Q_y^{-1}\{\alpha - \Sigma_y\}.$$

A straightforward computation obtains (38) with (40)–(41). □

Remarks. 1 The PFL control law implements an effective dynamic inverse model for the Lagrangian system with respect to the dynamic response of attitude coordinates from the generalized torques τ using state feedback so that the closed loop response is linear and decoupled. Linearity of the input-output response is achieved relative to a nominal model of the system.

More importantly, the PFL control functions to decouple the dynamics associated with the u coordinates from the (attitude) γ coordinates. In Singh (1987) attention is directed towards implementation of feedback linearizing compensation by consideration of the invertibility of a decoupling matrix. In our construction, invertibility of the decoupling matrix required for PFL attitude control is guaranteed and model assumptions which affect decoupling are explicit in the Lagrangian formulation.

2 For the SCOLE design challenge, the natural choice of controls for attitude slewing of the shuttle-mast-reflector system is the three independent shuttle-body torques generated by the shuttle orbiter attitude control system. In this case, Eqs. (40)-(41) simplify. Comparing (35)-(36) with (30) we see that $G_S^T = [G_\omega, G_u] = [I_3, 0]$. Thus the system decoupling matrix:

$$[G_\omega - NM_u^{-1}G_u] = I_3 \tag{43}$$

is trivially invertible in these coordinates. From (38)-(41) and (14) we see that the effective inverse system inertia matrix for PFL control of system attitude is:

$$NM_u^{-1}N^T - M_\omega = I_{\omega\eta}I_{\eta\eta}^{-1}I_{\omega\eta}^T + J_{\omega\xi}J_{\xi\xi}^{-1}J_{\omega\xi}^T - I_{sys}$$

where $I_{sys} = (I_S + I_R + I_{\omega\omega} + J_{\omega\omega})$. Thus from (41) we see that for SCOLE system model,

$$\mathcal{B}(\eta,\gamma_S) = [I_{\omega\eta}I_{\eta\eta}^{-1}I_{\omega\eta}^T + J_{\omega\xi}J_{\xi\xi}^{-1}J_{\omega\xi}^T - I_{sys}]\Gamma^{-1}(\gamma_S) \tag{44}$$

Likewise from (29)-(31) we can identify the additional PFL terms in (40):

$$\begin{pmatrix}\phi_\omega\\ \phi_u\end{pmatrix} = -\begin{bmatrix}B_{\omega\omega} & B_{\omega u}\\ B_{u\omega} & B_{uu}\end{bmatrix}\begin{pmatrix}\omega\\ \dot{u}\end{pmatrix} - \begin{bmatrix}0 & 0\\ 0 & K_{mast}\end{bmatrix}\begin{pmatrix}\gamma\\ u\end{pmatrix} + \begin{bmatrix}G_{R\omega}\\ G_{Ru}\end{bmatrix}\bar{f}_R \tag{45}$$

where $G_R^T = [G_{R\omega}, G_{Ru}]$.

3 Note that the PFL attitude control for shuttle includes feedforward from active structure control forces $\bar{f}_R$ acting on the mast/reflector system (re. last term in Eq. 45). The PFL control functions to decouple the attitude slewing control for the shuttle-mast-reflector system from the active stabilization and structural alignment control of the mast/reflector system flexure. This will be required to satisfy specifications on settling of the rf LOS of the SCOLE system after slewing.

Active Structural Alignment Control. An important feature of the PFL attitude control of a flexible spacecraft of the type exemplified by the SCOLE configuration is the decoupling of the attitude control system from active structural alignment control through feedforward. The SCOLE design challenge directs attention to the problem of rapid spacecraft slewing and subsequent settling of the LOS by active stabilization of the mast/reflector system using controls $\bar{f}_R^T = [f_R^T, \tau_R^T]$ at the reflector.

In the present study we introduce active structural control to enhance damping of the mast/reflector system (and thus LOS settling) using a constant gain rate feedback control at the reflector. We assume collocated deformation rate (i.e., strain rate) measurements at the reflector. The structure control law for active damping enhancement is: $\bar{f}_R = -F_r y_s$ where $y_s^T = [\dot{\eta}^T(l), \dot{\xi}^T(l)]$. The 5 × 5, positive definite, rate damping gain matrix F_r is chosen via a least squares approximation to a desired modal damping using the procedure suggested in Joshi (1989).

Adaptive PFL Attitude Control. A principle concern in the practical application of PFL control laws arises due to concern for robustness to model uncertainty. Since PFL control laws function by implementation of an online inverse model, robustness to residual or parasitic system dynamics which may have been neglected in the available plant model, as well as robustness to parametric uncertainty-are of concern. In Bennett et al. (1990a) we give specific considerations for robust stabilization of flexible structures using PFL controls. In the present paper we focus attention on parametric uncertainty of the nominal plant model arising due to uncertainty in the stiffness of the mast/reflector system. Our approach is based on application of a modified Model Reference Adaptive Control (MRAC) scheme for PFL which follows from constructions described in Taylor et al. (1988) and Akhrif (1989).

The idea is to explicitly recognize parametric uncertainty in the plant model (35)-(36):

$$M_\omega(\vartheta)\dot{\omega} + N(\vartheta)\ddot{u} + \phi_\omega(\vartheta) = G_\omega\tau \tag{46}$$

$$M_u(\vartheta)\ddot{u} + N(\vartheta)^T\dot{\omega} + \phi_u(\vartheta) = G_u\tau \tag{47}$$

where ϑ is a k-vector of unknown system parameters (e.g., inertias, stiffness, etc.). At design time one normally assumes certain nominal values for the parameters and proceeds to implement a PFL control based on the nominal model. The basis for MRAC approach is to construct a convergent parameter estimator which provides online parameter estimates, $\hat{\vartheta}$. With explicit recognition of the parametric dependence, the PFL control law (38) assumes the form

$$\tau(\vartheta) = \mathcal{A}_\vartheta(\gamma,\dot{\gamma},u,\dot{u}) + \mathcal{B}_\vartheta(\gamma,\dot{\gamma},u,\dot{u})\alpha. \tag{48}$$

Then $\hat{\tau} = \tau(\hat{\vartheta})$ is the "certainty equivalent" PFL control (Taylor et al., 1989) implemented based on available online estimates of the parameters.

A convenient design procedure which obtains both a convergent parameter adaptive MRAC scheme and guarantees stable closed loop operation is available under two additional assumptions. First, that for any allowed value of ϑ the control $\tau(\vartheta)$ is a valid PFL control which obtains the decoupling shown in (39). This is assured by our construction of $\mathcal{A}$, $\mathcal{B}$. Second, for any estimate $\hat{\vartheta}$, it is required that

$$\ddot{\gamma} = \alpha + \Psi(\gamma,\dot{\gamma},u,\dot{u})(\vartheta - \hat{\vartheta}), \tag{49}$$

i.e., that the residual attitude acceleration of the regulated attitude variables is *linear in the parameter estimation error* (Taylor et al., 1989). This condition depends on how the parameters enter the governing dynamical equations.

The MRAC PFL design uses a reference model based on the decoupled response of the regulated variables as follows. Let $z^T = [y^T, \dot{y}^T]$. The resulting decoupled dynamics obtained from ideal PFL control, i.e., (39), can be written in state space form as:

$$\dot{z} = Az + B\alpha$$

where

$$A = \begin{bmatrix}0 & I_m\\ 0 & 0\end{bmatrix}, \quad B = \begin{bmatrix}0\\ I_m\end{bmatrix}.$$

The states of the reference model represent the ideal attitude and velocity response of the decoupled system obtained by PFL compensation.

MRAC design proceeds by selecting a servo control law:

$$\begin{aligned}\alpha &= K_p(\gamma_S - z_1) + K_r(\dot{\gamma}_S - z_2) + \gamma_c\\ &= Ke + \gamma_c\end{aligned} \tag{50}$$

with model following error $e^T = [\gamma_S^T - z_1^T, \dot{\gamma}_S^T - z_2^T]$ and where γ_S, γ_c are respectively the actual and commanded attitude position coordinates, $z = [z_1, z_2]$ are the states of the reference model. The Gibbs vector components and rates are obtained from available measurements on the shuttle, and $K = [K_p, K_r]$ is a $m \times 2m$ matrix with diagonal submatrices of position and rate gains which are chosen to obtain stable and desired slewing transient response to attitude commands for the decoupled model.

Theorem. Under the above assumptions on the structure of the parametric model uncertainty for the Lagrangian system

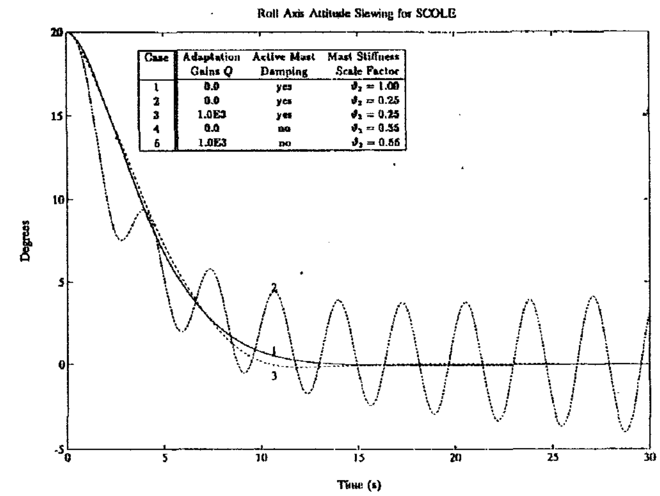

Fig. 2 Attitude slewing for SCOLE with active structure damping

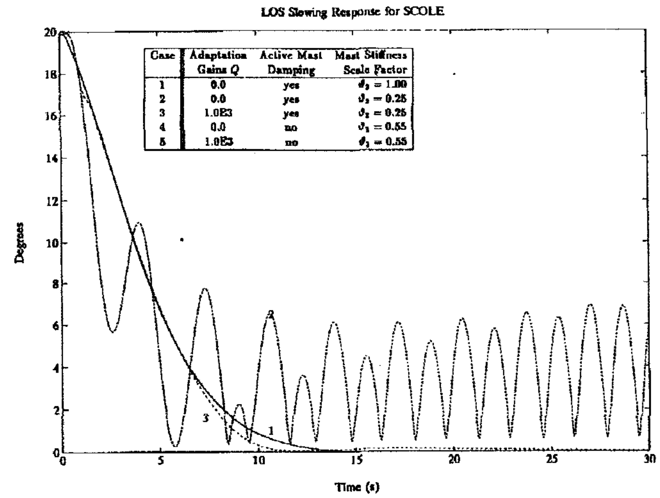

Fig. 4 LOS slewing for SCOLE with active structure damping

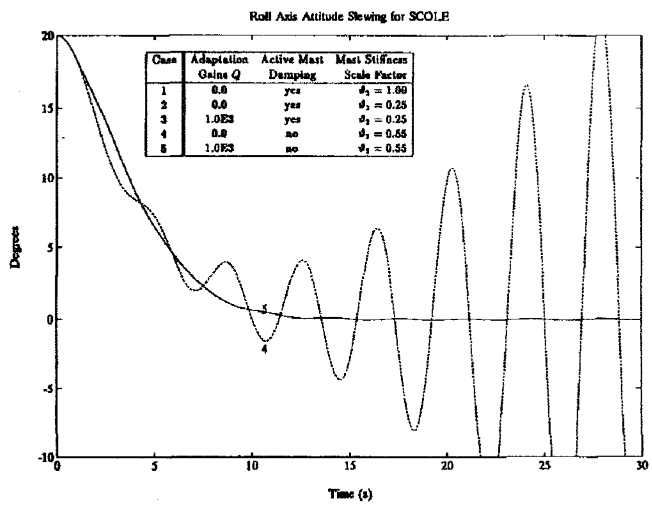

Fig. 3 Attitude slewing for SCOLE without active structure damping

(46)–(47), the PFL MRAC system including PFL control (48), the model-following law (50), with (40), (41) appropriately defined in terms of the parametric model uncertainty, together with the stabilized parameter update strategy:

$$\dot{\hat{\vartheta}} = Q\Psi^T(z,u,\hat{r})B^TPe \tag{51}$$

will provide asymptotic regulation of the tracking error $e \to 0$ for any symmetric positive definite matrix P such that,

$$(A+BK)^TP + P(A+BK) = -I$$

and any positive definite, $k \times k$ *adaptation gain* matrix Q. The *regressor* matrix Ψ, identified in (49), is a nonlinear function of the model states and control commands. (See Taylor et al. (1989) and Akhrif (1989) for details.) □

Structural Uncertainty Model for Spacecraft Attitude Slewing. The SCOLE design for adaptive PFL attitude slewing and LOS pointing considered herein focuses attention on uncertainty arising due to variation in the stiffness and damping properties of the mast/reflector system. To simplify the exposition we introduce parametric model uncertainty, $\vartheta^T = [\vartheta_1, \vartheta_2]$: an uncertain scaling of the mast stiffness and damping matrices:

$$B_{\text{mast}} := \vartheta_1 B^{\circ}_{\text{mast}} \tag{52}$$

$$K_{\text{mast}} := \vartheta_2 K^{\circ}_{\text{mast}} \tag{53}$$

where B°_{mast}, K°_{mast} are the nominal damping and stiffness matrices available at design time.

In an appendix we show that with the above assumptions on parametric model uncertainty for SCOLE system dynamics the assumption on linearity of the PFL residual is satisfied and we obtain a 3×2 regressor of the form:

$$\Psi(\eta,\dot{\eta},\xi,\dot{\xi}) = B^{-1}[I_{\omega\eta}I_{\eta\eta}^{-1}, J_{\omega\xi}J_{\xi\xi}^{-1}]\left\{\begin{bmatrix} B_{\eta\eta} & B_{\xi\eta}^T \\ B_{\xi\eta} & B_{\xi\xi}\end{bmatrix}\begin{pmatrix}\dot{\eta} \\ \dot{\xi}\end{pmatrix} + \begin{bmatrix} K_{\eta\eta} & K_{\xi\eta}^T \\ K_{\xi\eta} & K_{\xi\xi}\end{bmatrix}\begin{pmatrix}\eta \\ \xi\end{pmatrix}\right\} \tag{54}$$

with

$$\mathfrak{B}(\eta,\gamma_S)^{-1} = \Gamma(\gamma_S)[I_{\omega\eta}I_{\eta\eta}^{-1}I_{\omega\eta}^T + J_{\omega\xi}J_{\xi\xi}^{-1}J_{\omega\xi}^T - I_{\text{sys}}]^{-1} \tag{55}$$

Summary of Simulated Attitude Slewing

Simulation studies were completed for the SCOLE model given above using the system parameters as given in Taylor and Balakrishnan, (1984) (cf. Table 1). The simulation model used in this study was a reduced model including the first 13 modes of a model based on a FEM approximation of the deformation of the mast/reflector system using 21 equally spaced node points. The mode frequencies obtained compared favorably with the NASTRAN analysis reported in Fisher (1989). Attitude control was designed for the SCOLE system response to obtain ideal decoupled multiaxis slewing response consistent with the shuttle acceleration limits for a nominal 20 degree slew. The individual axis slewing gains were $K_p = -.1790I_3$, $K_r = -.7615I_3$.

We compared five cases to illustrate the performance potential from MRAC PFL compensation for SCOLE slewing using a standard maneuver of 20° in roll and 1° in both pitch and yaw. The conditions for the five simulation cases are now summarized in Figs. 2–5. The first case shows the performance predicted by the model-based PFL control (i.e., when the correct stiffness and damping properties of the system are known at design time). The second and third cases contrast the effectiveness of the adaptive PFL control law when the simulation model had a 75% reduction in mast stiffness from the model available at design time. Cases 2–3 include active damping enhancement of the mast/reflector deflection implemented using constant deformation (strain) rate feedback at the reflector. For simulation cases 1–3 we plot the shuttle attitude slewing response in Fig. 2 and the LOS response in Fig. 3.

In the simulation cases 4–5 we implement the PFL attitude control for slewing the shuttle without the active damping enhancement for the mast/reflector system. The simulated time responses in shuttle attitude and LOS are shown in Figs. 4 and

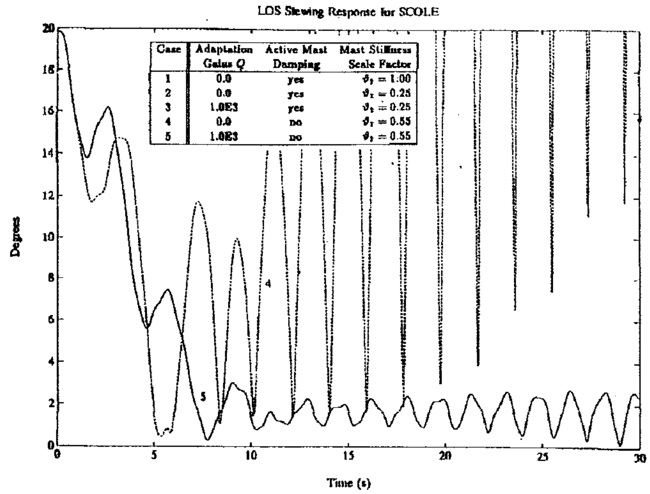

Fig. 5 LOS slewing for SCOLE without active structure damping

5. Note that the MRAC PFL attitude control provides stable regulation of the shuttle attitude with a reduction of mast stiffness of slightly better than 45% without active structural damping but does not guarantee stable settling of the LOS. This is true despite the fact that the simulation model involves a dissipative mast/reflector structure. Also note that without MRAC the constant gain PFL control does not provide stable response with the same level of stiffness uncertainty in the nominal design model.

Conclusions

We have demonstrated the potential for nonlinear adaptive control for multiaxis, large angle slewing and LOS pointing of multibody models for spacecraft with elastic structural interactions by consideration of the SCOLE system. Our modeling and control design approach is comprehensive and includes detailed development of system dynamics, decomposition of control authority, and design of decoupling and PFL control laws for attaining the control objectives. The approach to adaptive PFL control is shown to be effective for a spacecraft with significant uncertainty in the appendage flexure dynamics. The importance of integrated design including active, low authority damping enhancement for robust slewing and pointing control underlies the potential significance of nonlinear dynamics in understanding and compensating for Control-Structure-Interactions in multi-flex-body systems.

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Dedication

This paper is dedicated to the memory of Professor Thomas A. W. Dwyer.

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APPENDICES

System Inertia for SCOLE Model

The expansion of the kinetic energy expressions results in the following terms appearing in the system inertia matrix.

$$I_{\omega\omega}(\bar{\eta}) = \begin{bmatrix} \bar{\eta}_2^T N_{\eta\eta} \bar{\eta}_2 + \sigma & -\bar{\eta}_1^T N_{\eta\eta} \bar{\eta}_2 & -\bar{\eta}_1^T N_\eta \\ -\bar{\eta}_1^T N_{\eta\eta} \bar{\eta}_2 & \bar{\eta}_1^T N_{\eta\eta} \bar{\eta}_1 + \sigma & -\bar{\eta}_2^T N_\eta \\ -\bar{\eta}_1^T N_\eta & -\bar{\eta}_2^T N_\eta & \bar{\eta}_2^T N_{\eta\eta} \bar{\eta}_2 + \bar{\eta}_1^T N_{\eta\eta} \bar{\eta}_1 \end{bmatrix}, \tag{56}$$

where

$$N_{\eta\eta} = \int_0^l \rho A \Phi(z) \Phi^T(z) dz + m_R \Phi(l) \Phi^T(l), \tag{57}$$

$$N_\eta^T = \int_0^l \rho A z \Phi^T(z) dz + m_R l \Phi^T(l), \tag{58}$$

$$\sigma = \int_0^l \rho A z^2 dz = \frac{\rho A l^3}{3} + m_R l^2. \tag{59}$$

Also,

$$I_{\omega\eta}(\bar{\eta}) = \begin{bmatrix} 0 & -N_\eta^T \\ N_\eta^T & 0 \\ -\bar{\eta}_2^T N_{\eta\eta} & \bar{\eta}_1^T N_{\eta\eta} \end{bmatrix}, \quad I_{\eta\eta} = \begin{bmatrix} N_{\eta\eta} & 0 \\ 0 & N_{\eta\eta} \end{bmatrix}, \tag{60}$$

$$J_{\omega\omega} = \int_0^l \rho I(z) dz \tag{61}$$

$$J_{\omega\xi}=\begin{bmatrix}0 & N_l^T & N_\phi^T\\ 0 & N_\theta^T & N_l^T\\ N_\psi^T & 0 & 0\end{bmatrix},\quad J_{\xi\xi}=\begin{bmatrix}N_{\theta\theta} & 0 & 0\\ 0 & N_{\phi\phi} & N_{\phi\psi}\\ 0 & N_{\phi\psi} & N_{\psi\psi}\end{bmatrix},\quad (62)$$

are obtained in terms of the following integral expressions:

$$N_l=I_{R,xy}\Phi^T(l) \quad (63)$$

$$N_\psi^T=\int_0^l \rho I_{zz}\Phi^T(z)dz+I_{R,zz}\Phi^T(l) \quad (64)$$

$$N_\theta^T=\int_0^l \rho I_{yy}\Phi^T(z)dz+I_{R,yy}\Phi^T(l) \quad (65)$$

$$N_\phi^T=\int_0^l \rho I_{xx}\Phi^T(z)dz+I_{R,xx}\Phi^T(l) \quad (66)$$

$$N_{\phi\phi}=\int_0^l \rho I_{xx}\Phi(z)\Phi^T(z)dz+I_{R,xx}\Phi(l)\Phi^T(l) \quad (67)$$

$$N_{\theta\theta}=\int_0^l \rho I_{yy}\Phi(z)\Phi^T(z)dz+I_{R,yy}\Phi(l)\Phi^T(l) \quad (68)$$

$$N_{\psi\psi}=\int_0^l \rho I_{zz}\Phi(z)\Phi^T(z)dz+I_{R,zz}\Phi(l)\Phi^T(l) \quad (69)$$

$$N_{\phi\psi}=I_{R,xy}\Phi(l)\Phi^T(l) \quad (70)$$

Expansion of the Potential Energy Function

The stiffness coefficient matrices are obtained by FEM expansion of the potential energy expression as

$$K_{\eta\eta}=\text{diag}\{K_\eta,K_\eta\} \quad (2N\times 2N) \quad (71)$$

$$K_{\xi\xi}=\text{diag}\{K_\psi,K_\theta,K_\phi\} \quad (3N\times 3N) \quad (72)$$

$$K_{\xi\eta}=\begin{bmatrix}0 & 0\\ K_{\theta\eta} & 0\\ 0 & K_{\theta\eta}\end{bmatrix} \quad (3N\times 2N) \quad (73)$$

The following $N\times N$ matrices are obtained by direct evaluation of the integrals obtained from the FEM expansions:

$$K_\psi=\int_0^l \{\kappa_3 GI_{zz}\Phi_z(z)\Phi_z^T(z)\}dz \quad (74)$$

$$K_\theta=\int_0^l \{EI_{yy}\Phi_z(z)\Phi_z^T(z)+\kappa_1 GA\Phi(z)\Phi^T(z)\}dz \quad (75)$$

$$K_\phi=\int_0^l \{EI_{xx}\Phi_z(z)\Phi_z^T(z)+\kappa_1 GA\Phi(z)\Phi^T(z)\}dz \quad (76)$$

$$K_\eta=\int_0^l \{\kappa_1 GA\Phi_z(z)\Phi_z^T(z)\}dz \quad (77)$$

$$K_{\theta\eta}=-\int_0^l \{\kappa_1 GA\Phi(z)\Phi_z^T(z)\}dz \quad (78)$$

Dissipation Function Expansions. For simplicity we take $\Xi_i=\zeta_i I_3$ and, as for the stiffness terms, assume the damping matrices have the form,

$$B_{\eta\eta}=\text{diag}\{B_\eta,B_\eta\} \quad (2N\times 2N) \quad (79)$$

$$B_{\xi\xi}=\text{diag}\{B_\psi,B_\theta,B_\phi\} \quad (3N\times 3N) \quad (80)$$

$$B_{\xi\eta}=\begin{bmatrix}0 & 0\\ B_{\theta\eta} & 0\\ 0 & B_{\theta\eta}\end{bmatrix} \quad (3N\times 2N). \quad (81)$$

The FEM expansions obtain the matrix dissipation coefficients as:

$$B_\eta=\int_0^l \zeta_1\Phi(z)\Phi^T(z)+\zeta_3\Phi_z(z)\Phi_z^T(z)dz \quad (82)$$

$$B_\psi=B_\theta=B_\phi=\int_0^l (\zeta_2+\zeta_3)\Phi(z)\Phi^T(z)+\zeta_4\Phi_z(z)\Phi^T(z)dz \quad (83)$$

$$B_{\theta\eta}=-\zeta_3\int_0^l \Phi^T(z)\Phi_z(z)dz. \quad (84)$$

Derivation of Regressor for Adaptation

To identify the regressor matrix for estimation of the stiffness and damping scaling factors we start by writing the closed loop system dynamics with certainty equivalent PFL control $\hat{\tau}=\tau(\hat{\vartheta})$ in the form:

$$\dot{z}=Az+B\alpha+B\mathcal{B}^{-1}(\hat{\tau}-\tau^*)$$

where $\tau^*=\tau(\vartheta)$ the correct PFL control. To obtain the expression for the regressor we direct attention to the PFL residual which we seek to rewrite as,

$$\mathcal{B}^{-1}(\hat{\tau}-\tau^*)=\Psi(\hat{\vartheta}-\vartheta).$$

Given the PFL control (48) and the parametric model dependence (52)–(53) and (44) we see that PFL control torque residual is independent of B. From (40) and (43) we obtain

$$\hat{\tau}-\tau^*=\hat{\mathcal{C}}-\mathcal{C}=[I,-NM_u^{-1}]\begin{pmatrix}\hat{\phi}_\omega-\phi_\omega\\ \hat{\phi}_u-\phi_u\end{pmatrix}$$

then from (45) and (31)

$$\hat{\mathcal{C}}-\mathcal{C}=-\text{N}M_u^{-1}\{(\vartheta_1-\hat{\vartheta}_1)B_{\text{mast}}\dot{u}+(\vartheta_2-\hat{\vartheta}_2)K_{\text{mast}}u\}$$

$$=[-NM_u^{-1}B_{\text{mast}}\dot{u},-NM_u^{-1}K_{\text{mast}}u]\begin{pmatrix}\vartheta_1-\hat{\vartheta}_1\\ \vartheta_2-\hat{\vartheta}_2\end{pmatrix}$$

Finally, we identify the 3 × 2 regressor in the form (54).